(2) $r(1-\cos\theta) = 2a$

$$\frac{2a}{r} = (1-\cos\theta)$$

differentiating w.r.t $\theta$, we get

$$-\frac{2a}{r^2}\cdot\frac{dr}{d\theta} = \sin\theta$$

$$\boxed{\frac{dr}{d\theta} = -\frac{r^2\sin\theta}{2a}}$$

We have. $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left[\frac{dr}{d\theta}\right]^2$

$$= \frac{1}{r^2} + \frac{1}{r^4}\left[\frac{r^2\sin\theta}{2a}\right]^2$$

$$= \frac{1}{r^2} + \frac{1}{r^4}\left[\frac{r^4\sin^2\theta}{4a^2}\right] = \frac{1}{r^2} + \frac{\sin^2\theta}{4a^2}$$

$$\boxed{\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{4a^2}(1-\cos^2\theta)} \quad -(1)$$

Given. $r(1-\cos\theta) = 2a$

$$\frac{2a}{\cancel{r}} = r(1-\cos\theta)$$

~~1 - cocos~~ $(1-\cos\theta) = \frac{2a}{r}$

$$\cos\theta = 1 - \frac{2a}{r} = \boxed{\frac{r-2a}{r}} \quad -(2)$$

(2) in (1). we get

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{4a^2}(1-\cos^2\theta)$$

$$= \frac{1}{r^2} + \frac{1}{4a^2}\left[1-\left(\frac{r-2a}{r}\right)^2\right]$$

$$= \frac{1}{r^2} + \frac{1}{4a^2}\left[1-\frac{(r-2a)^2}{r^2}\right]$$

$$= \frac{1}{r^2} + \frac{1}{4a^2}\left(\frac{r^2-[r^2+4a^2-4ar]}{r^2}\right)$$

$$= \frac{1}{r^2} + \frac{1}{4a^2}\left[\frac{4ar-4a^2}{r^2}\right]$$

$$= \frac{1}{r^2} + \frac{1}{4a^2}\left[\frac{4a[r-a]}{r^2}\right]$$

$$= \frac{1}{r^2} + \frac{r-a}{ar^2}$$

$$\boxed{\frac{1}{p^2} = \frac{1}{r^2} + \frac{r-a}{ar^2}}$$